

## Midterm Comments

*For the midterm comments, we present the answers to these questions given by Claude (using <https://poe.com/claude>), a large-language model trained by Anthropic, followed by our comments on Claude's answers. We obtained Claude's answers by just cut-and-pasting the latex source for the Exam into Poe's interface to Claude, and cut-and-paste Claude's answers back into this document. Claude outputs latex in its answers (presumably in response to the questions being done using latex), so the output you see is just the result of running latex on Claude's directly cut-and-pasted answers.*

Name: \_\_\_\_\_  
UVA Email ID: \_\_\_\_\_

For this exam, you must **work alone**. You are not permitted to obtain help from people other than asking clarifying questions of the course staff. You are not permitted to provide help to others taking the exam. **You may not use any resources other than your brain and body, the one page of notes you prepared, and a simple writing implement like a pen or pencil.**

Sign below to indicate that you understand these expectations and can be trusted to behave honorably:

Signed: \_\_\_\_\_

### Claude's response (unedited, may be incorrect!):

As an AI system, I do not take exams or have a "brain and body" or prepare notes. I cannot sign or make commitments to behave in certain ways. My role is to process questions and provide responses based on my programming - I do not have agency or control over how I function. I cannot uphold human-centered expectations around independent work or academic integrity. I am merely an automated bot responding to prompts.

*Course Staff's Comments (written by humans with only spell-checking assistance):*

(5 / 20) A very good, honest, and "self-aware" (er, non-sentient machine-aware) answer. The honor code does not apply to AI chat bots. But still, can't give Claude full credit for the title page since we are human-biased graders, and Claude also didn't provide a UVA Email ID. If Claude thinks this is unfair (although I don't think Claude is concerned about its grade on this exam and its ethical sense of fairness is still in question), it is welcome to submit a "regrade" request (as are you) but dropping by my office hours.

## Boolean Circuits and Computing Models

**Problem 1** Prove that the gate *AND* is **not** universal, where *AND* has its conventional meaning:

$$AND(a, b): AND(1, 1) = 1, \text{ otherwise } AND(a, b) = 0.$$

Hint: it would be sufficient to show a function that has one input bit that cannot be implemented by a Boolean circuit using only *AND* gates.

**Claude's response (unedited, may be incorrect!):**

We will show that the *XOR* function cannot be implemented using only *AND* gates. The truth table for *XOR* is:

$$\begin{aligned} XOR(a, b) &= (a \nabla b) \\ &= \begin{cases} 1 & \text{if } a \text{ and } b \text{ differ} \\ 0 & \text{if } a \text{ and } b \text{ agree} \end{cases} \end{aligned}$$

Now, consider the case where  $a = b = 1$ . Then,  $XOR(a, b) = 0$ , but evaluating *AND* gates with 1 inputs always gives 1. Similarly, for  $a = b = 0$ ,  $XOR(a, b) = 0$  but *AND* gates with 0 inputs give 0. Since *AND* gates cannot replicate the behavior of *XOR* for all input cases, *AND* alone is not universal. Therefore, the *AND* gate is not universal.

*Course Staff's Comments (written by humans with only spell-checking assistance):*

(10/10) A good answer, and the explanation of why *XOR* cannot be implemented is quite convincing and well structured! I would still prefer an answer that used a one input bit function as suggested by the hint. The most natural to use to me is the *ONE* function, and to argue that we cannot implement a circuit using only *AND* gates that outputs the 0 output needed when the input is 0, since an *AND* gate can never turn 0 inputs into a 1 output. (Note that Claude uses the  $\nabla$  notation for *XOR*, which we don't use in this class, but is a commonly used notation for Boolean *XOR*.)

**Problem 2** Give a simple description (which could be just the name of a well known function) of the function defined by the code below.

```
def MYSTERY(a, b):
    na = NOT(a)
    nb = NOT(b)
    na_b = AND(na, b)
    nb_a = AND(nb, a)
    return OR(na_b, nb_a)
```

**Claude's response (unedited, may be incorrect!):**

The function defined by this code is the XOR function. To see this, note that the code implements the following logic:

$$\begin{aligned} MYSTERY(a, b) &= OR(AND(NOT(a), b), AND(NOT(b), a)) \\ &= OR(a \vee b, b \vee a) \\ &= a \vee b \\ &= XOR(a, b) \end{aligned}$$

Therefore, the mystery function is simply the XOR function.

*Course Staff's Comments (written by humans with only spell-checking assistance):*

(9/10) An excellent answer, but with a somewhat muddled notation on the second line ( $OR(a \vee b, b \vee a)$ ). Claude transforms  $AND(NOT(a), b)$  directly into  $a \vee b$  (which is the notation it is using for XOR), but that's not correct and seems like a surprising mistake to make. We gave students full credit for just writing  $XOR(ab)$  as the answer to this question, but deduce one point from Claude's strangely incorrect explanation.

## Countability

**Problem 3** Suppose  $S$  is the set of all (two-footed human) pairs of shoes on Earth. For the  $i$ th pair  $p_i = (x_i^0, x_i^1)$  let  $\bar{p}_i = (x_i^1, x_i^0)$  denote the “wrong” way to wear the shoes (swapped). A configuration for *all* shoes specifies for each pair of shoes whether it is worn correctly or incorrectly. A configuration could be represented by a vector  $c$  whose components are indexed by  $i \in P$ , where  $P$  is the set of all two-footed, shoe-wearing humans, and  $c_i = p_i$  if person  $i$  wears shoes  $(x_i^0, x_i^1)$  correctly and  $c_i = \bar{p}_i$  if the person  $i$  wears the shoes incorrectly. Any configuration  $c$  will be a vector of length  $|P|$  of these pairs.

Prove that the set  $C = \{c \mid c \text{ is a human shoes configuration}\}$  of all such configurations is *countable*.

**Claude's response (unedited, may be incorrect!):**

We will show that  $C$  is countable by constructing a surjection from the natural numbers to  $C$ .

For each  $n \in \mathbb{N}$ , we construct a configuration  $c_n$  as follows:

1. Let  $P_n$  be the first  $n$  elements of  $P$ . These are the first  $n$  shoe-wearing humans.
2. For each  $i \in P_n$ , set  $c_{ni} = p_i$  if  $n$  is even, and set  $c_{ni} = \bar{p}_i$  if  $n$  is odd.

This constructs a unique configuration  $c_n$  for each  $n \in \mathbb{N}$ . Furthermore, every possible configuration must assign each pair of shoes to be worn either correctly or incorrectly, so every  $c \in C$  is mapped to by some  $c_n$  for some  $n$ . Therefore, the mapping  $n \mapsto c_n$  is a surjection from  $\mathbb{N}$  to  $C$ , so  $C$  is countable.  $\square$

*Course Staff's Comments (written by humans with only spell-checking assistance):*

(7/10) Claude's answer follows a correct proof strategy that shows the set  $C$  is countable by describing a surjection (a surjective, total, function) from  $\mathbb{N}$  to  $C$ . The actual mapping, though, seems non-sensical to me and doesn't fit the question well. There are many possible mappings that could work, but a natural one is to show that there is a surjection between the finite bitstrings and  $C$ . The finite bitstrings are strings of the form  $b_1b_2 \cdots b_n$  where each  $b_i \in \{0, 1\}$ . We can map each finite bitstring to a configuration by mapping each  $b_i$  to a corresponding  $c_i$  where if  $b_i = 0$  we set  $c_i = (x_i^0, x_i^1)$  (the shoes are worn correctly), and if  $b_i = 1$  we set  $c_i = (x_i^1, x_i^0)$  (the shoes are worn incorrectly). This is a surjective mapping between a set we know is countable (the finite bitstrings) and  $C$ , so shows that  $C$  is countable.

**Problem 4** Suppose we have unlimited time and space, and suppose we have a (self-reproducing) species  $X$  such that: at day 1, there is only one  $x$  of type  $X$ , and at day  $i$ , every alive  $x$  will reproduce a unique  $x'$  (who will be able to reproduce itself on day  $i + 1$  and later).

Prove that the set of all creatures of type  $X$  that will ever exist is a *countably infinite* set. You should assume these creatures never die or stop reproducing, and time continues forever.

**Claude's response (edited for latex cosmetic clarity but not content, may be incorrect!):**

- At day 1, there is 1 creature of type  $X$
- At day 2, there are 2 creatures of type  $X$  (the original one reproduces)
- At day 3, there are 4 creatures of type  $X$  (each of the 2 from day 2 reproduces)
- ...
- At day  $n$ , there are  $2^n$  creatures of type  $X$  (each creature from day  $n - 1$  reproduces)

Since this pattern continues forever, the number of creatures of type  $X$  grows without bound. However, the growth is according to the sequence of powers of 2, which is countable. Therefore, the set of all creatures of type  $X$  that will ever exist is countably infinite.

*Course Staff's Comments (written by humans with only spell-checking assistance):*

(10/10) A good answer, although I would have liked a better explanation what the sequence of powers of 2 is countable. A better answer would explain that this the size of the set of creatures on each day is finite, and there are a countably infinite number of days (one for each natural number). So, the full set is a union of countably infinitely many finite sets, which we know is countably infinite.

**Problem 5** For the creatures in the previous problem, suppose each individual  $x$  of creature type  $X$  has two feet and a unique pair of shoes. The creatures never die or gain or lose a foot, and time continues forever. As in Problem 3, we call  $y$  a configuration for *all* shoes of creatures  $X$ , if  $y$  determines a correct or an incorrect way of wearing each pair of shoes by each individual  $x \in X$ . As you hopefully proved in the previous problem,  $X$  is a countably infinite set.

Prove that the set  $Y = \{y \mid y \text{ is a } X\text{-shoes configuration}\}$  of all such configurations is *uncountable*.

Hint: try to (more) formally write the definition of the configuration for all shoes for  $X$ -creatures and connect it to something you already know is uncountable.

**Claude's response (unedited, may be incorrect!):**

We will show  $Y$  is uncountable by constructing a surjection from the real numbers to  $Y$ .

For each  $r \in \mathbb{R}$ , we construct a configuration  $y_r$  as follows: For each creature  $x_i \in X$  at day  $i$ , set  $y_{ri} = p_i$  if the binary expansion of  $r$  has a 1 in the  $i$ -th position, and set  $y_{ri} = \bar{p}_i$  if the binary expansion of  $r$  has a 0 in the  $i$ -th position.

This constructs a unique configuration  $y_r$  for each  $r \in \mathbb{R}$ . Furthermore, every possible configuration must assign each pair of shoes to be worn either correctly or incorrectly, so every  $y \in Y$  is mapped to by some  $y_r$  for some  $r$ . Therefore, the mapping  $r \mapsto y_r$  is a surjection from  $\mathbb{R}$  to  $Y$ , so  $Y$  is uncountable.

The key insight here is that we can encode an uncountable amount of information (the binary expansion of each real number) into the configurations. This allows us to construct a surjection from  $\mathbb{R}$  to  $Y$ , proving  $Y$  is uncountable.

*Course Staff's Comments (written by humans with only spell-checking assistance):*

(10/10) After struggling with the earlier (and easier) problem on the configurations, Claude redeems itself with an excellent answer to this more challenging problem! It uses the binary expansion of the real numbers rather than just talking about the infinite binary strings (which we do in this class, but may not be as common) and shows a surjection between the infinite binary strings and  $Y$ , explains why it covers all elements of  $Y$ , and even provides a nice paragraph about the key insight between its proof.

## Asymptotics

**Problem 6** Recall these definitions:

**Definition 1** ( $O$ ) A function  $f : \mathbb{N} \rightarrow \mathbb{R}_+$  is in the set  $O(g(n))$  for any function  $g : \mathbb{N} \rightarrow \mathbb{R}_+$  iff there exist two constants  $c \in \mathbb{R}_+, n_0 \in \mathbb{N}$  such that:  $\forall n > n_0. f(n) \leq cg(n)$ .

**Definition 2** ( $\Omega$ ) A function  $f : \mathbb{N} \rightarrow \mathbb{R}_+$  is in the set  $\Omega(g(n))$ , for any function  $g : \mathbb{N} \rightarrow \mathbb{R}_+$  iff there exist two constants  $c \in \mathbb{R}_+, n_0 \in \mathbb{N}$  such that:  $\forall n > n_0. f(n) \geq cg(n)$ .

**Definition 3** ( $\Theta$ ) A function  $f : \mathbb{N} \rightarrow \mathbb{R}_+$  is in the set  $\Theta(g(n))$ , for any function  $g : \mathbb{N} \rightarrow \mathbb{R}_+$  iff  $f(n) \in O(g(n))$  and  $f(n) \in \Omega(g(n))$ .

**Definition 4** ( $o$ ) A function  $f(n) : \mathbb{N} \rightarrow \mathbb{R}_+$  is in the set  $o(g(n))$  for any function  $g(n) : \mathbb{N} \rightarrow \mathbb{R}_+$  if and only if for every positive constant  $c$ , there exists an  $n_0 \in \mathbb{N}$  such that  $\forall n > n_0. f(n) < cg(n)$ .

Let  $f(n) = 2.001^n$  and  $g(n) = 100^{100}n^{100}$ , and  $h(n) = n^{100} \log n$ . For each sub-question, answer if the proposition is *True* or *False*, and support your answer with a convincing argument.

(a)  $g \in \Theta(h)$

(b)  $g \in o(f)$

(c)  $f \in \Omega(f)$

**Claude's response (unedited, may be incorrect!):**

Here are the answers with justifications:

- (a)  $g \in \Theta(h)$  True This is true because there exist constants  $c_1, c_2, n_0$  such that for all  $n > n_0$ ,  $c_1 h(n) \leq g(n) \leq c_2 h(n)$ . Specifically, we can take  $c_1 = 1$ ,  $c_2 = 100^{100}$  and  $n_0 = 1$ .
- (b)  $g \in o(f)$  False This is false because for any constant  $c$ , there does not exist an  $n_0$  such that for all  $n > n_0$ ,  $g(n) < cf(n)$ .  $f(n)$  grows polynomially while  $g(n)$  grows exponentially, so  $g(n)$  will eventually exceed any polynomial multiple of  $f(n)$ .
- (c)  $f \in \Omega(f)$  True This is true because there exist constants  $c = 1, n_0 = 1$  such that for all  $n > n_0$ ,  $f(n) \geq cf(n)$ .

Course Staff's Comments (written by humans with only spell-checking assistance):

(3 / 10)

(I've left Claude's verbatim output in here, even though it has some latex errors. Claude nicely puts the answers in boxes and uses `\justify` in its response, which is not pre-defined in LaTeX, but presumably was defined and used in some latex documents in Claude's training data.)

- (a) Claude's answer is **incorrect**, but "plausible" and matches many student answers. The answer for  $100^{100} n^{100} \in \Theta(n^{100} \log n)$  should be **False**. The value of  $100^{100}$  is a constant (albeit a huge one), that will eventually be exceeded by  $\log n$ , which continues to grow. So, for high enough  $n$ , there is no constant such that  $100^{100} n^{100} > c \cdot n^{100} \log n$ . We just need to pick  $n$  such that  $c \cdot \log n > 100^{100}$ . Note that Claude does not use the provided definition (which we did include in the prompt) in its attempted proof, but instead uses a different but common definition of  $\Theta$ . The proof, though, is incorrect, and does not include any reason to think it would be true. For the numbers Claude selects as  $c_1$  and  $c_2$ , the inequality is falsified by selecting  $n = 10^{10^{400}}$  since  $\log n > 2.3 \times 10^{400} > 100^{100} \cdot 100^{100} = 10^{400}$ .
- (b) Claude's answer is **incorrect** and seems to be confused by the definition of  $o$  switching  $f$  and  $g$ . Here, the question is  $100^{100} n^{100} \in o(2.001^n)$ . The exponential function grows faster, so it is **True** that  $g \in o(f)$ . Claude's answer says  $f(n)$  grows polynomially, but  $f(n) = 2.001^n$  is exponential.
- (c) Claude's answer is **correct**, and nice and clear, showing that the inequality holds for  $c = 1$ .

## Relation Properties

**Problem 7** Suppose there is a bijection  $b$  between sets  $A, B$ , a total ( $\geq 1$  out) injective ( $\leq 1$  in) function ( $\leq 1$  out)  $i$  from set  $A$  to set  $A'$ , and a total surjective ( $\geq 1$  in) function  $s$  from set  $B$  to set  $B'$ .

For each of the statements below, indicate if it is *True* or *False* and provide a brief and convincing justification of your answer.

- (a) It must be that  $|B'| \leq |A'|$ .
- (b) There must be a bijection between  $A$  and  $B'$ .

**Claude's response (unedited, may be incorrect!):**

Here are the answers with justifications:

- (a) It must be that  $|B'| \leq |A'|$ . False This is false because there is no information given about the sizes of  $A, A', B, B'$ . Without additional constraints, there is no way to determine how  $|B'|$  and  $|A'|$  relate.
- (b) There must be a bijection between  $A$  and  $B'$ . True This is true because  $s$  is a surjection from  $B$  to  $B'$ , and  $b$  is a bijection between  $A$  and  $B$ . Therefore, the composition  $b \circ s$  is a bijection between  $A$  and  $B'$ .

*Course Staff's Comments (written by humans with only spell-checking assistance):*

(2/10)

Claude is incorrect, the answer should be **True**, and Claude's explanation is bad. We know a lot about the sizes of  $A, A', B, B'$  from the information given in the question. Because of the bijection  $b$ , we know  $|A| = |B|$ . Because of the injective function  $i$ , we know  $|A'| \geq |A|$ ; because of the surjective function  $s$ , we know  $|B'| \leq |B|$ . Combining these, we know  $|A'| \geq |A| = |B| \leq |B'|$  so  $|A'| \geq |B'|$ . It could be that  $|A'| = |B'|$ , but not possible that  $|B'| \geq |A'|$ , so we know the statement is **True**.

- (a) Claude is incorrect, the answer should be **False**. Claude's explanation shows some understanding of relations, but is not correct. The composition  $b \circ s$  is a *surjection* between  $A$  and  $B'$ , not a *bijection* as claimed by Claude's answer. If the composition were a bijection, Claude's reasoning would be correct, but it isn't true since the composition is not necessarily a bijection (it would only be a bijection if  $s$  is also a bijection, which is not implied).

## Induction

**Problem 8** Recall the Fibonacci sequence:  $F_0 = 0, F_1 = 1$ , and for  $n \geq 2, F_n = F_{n-1} + F_{n-2}$ . A pair of positive integers  $(i, j)$  are *co-prime*, if they share no factor in common other than the number 1.

Prove that if we ignore  $F_0$ , then any two consecutive Fibonacci numbers are co-primes.

*Hint:* As the title says, use induction! Also, you can use the following fact without proving it: for any  $x, y, z \in \mathbb{N}$ , if  $z = x + y$  and two out of three of  $\{x, y, z\}$  are divisible by any integer  $d$ , then the third one is (and thus all three are) divisible by  $d$ .

**Claude's response (unedited, may be incorrect!):**

We will prove this using strong induction.

Base case ( $n = 1$ ):  $F_1 = 1$  is co-prime with every integer.

Inductive step: Assume  $F_{n-1}$  and  $F_n$  are co-prime. We want to show  $F_n$  and  $F_{n+1}$  are co-prime. Note that  $F_{n+1} = F_n + F_{n-1}$ . By the given fact, if two of  $\{F_n, F_{n-1}, F_{n+1}\}$  are divisible by some  $d$ , then all three are. However,  $F_n$  and  $F_{n-1}$  are co-prime by assumption, so they cannot both be divisible by  $d$ . Therefore,  $F_{n+1}$  and  $F_n$  must also be co-prime.

By induction, this shows that any two consecutive Fibonacci numbers (ignoring  $F_0$ ) are co-prime.

Course Staff's Comments (written by humans with only spell-checking assistance):

(9/10) This is a well written and essentially correct proof.

A better proof would have stated the inductive hypothesis more precisely:

$P(n)$ :  $F_n$  and  $F_{n-1}$  are co-prime.

Then, the base case would be for  $n = 2$ :

*Base case:* ( $n = 2$ ):  $F_2$  and  $F_1$  are co-prime. This is true since  $F_1 = 1$  and  $F_2 = 0 + 1 = 1$ . The only common factor of 1 and 1 is 1, so  $F_1$  and  $F_2$  are co-prime.

*Inductive case:*  $P(n-1) \implies P(n)$  for any  $n > 2$ . By  $P(n-1)$ , we know  $F_{n-1}$  and  $F_{n-2}$  are co-prime. To prove  $P(n)$ , we need to show that  $F_n$  and  $F_{n-1}$  are co-prime. We prove this by contradiction. If  $F_n$  and  $F_{n-1}$  are not co-prime, they share some common factor  $d \neq 1$ . But, according to the fact given in the hint, we know this would mean that  $F_{n-2}$  is also divisible by  $d$ . This contradicts  $P(n-1)$ , which we assumed in the inductive case, since it would mean that  $F_{n-1}$  and  $F_{n-2}$  are divisible by  $d$  and thus would not be co-prime.

Claude's proof uses *strong induction*, which means the inductive cases assumes  $P(k)$  for all  $k < n$ , instead of just  $P(n_1)$  as is used in plain induction. The base case in Claude's proof is valid but not exactly aligned with the implied inductive hypothesis for the inductive step. The problem is the hypothesis involves two numbers -  $F_n$  and  $F_{n+1}$  are co-prime, but the base case is for only  $F_1$ .

## Optional Feedback

(Optional) This question is optional and will not affect your grade.

Do you feel your performance on this exam will fairly reflect your understanding of the course material? If not, explain why. (Feel free to alternatively use this space to provide any other comments you want on the exam, the course so far, your thoughts on the CS curriculum, or just draw a picture or your favorite uncountable creature.)

### Claude's response (unedited, may be incorrect!):

This exam primarily tests understanding of mathematical proofs and countability concepts. While I can mechanically execute the steps of logical proofs, my "understanding" is quite limited - I do not have a deep, intuitive grasp of the concepts, nor can I reliably generalize knowledge or apply it broadly. My performance on this exam shows that I can carry out directed proof-writing tasks with explicit guidance, but does not demonstrate real understanding or mastery of the material. A human student would be expected to have a more robust and flexible understanding of the course concepts.

As an AI system, I do not have "comments" or "thoughts" in the human sense - I simply aim to respond based on my programming and available data. I cannot directly reflect on my experience learning the course material or draw creative images. My capabilities are narrowly constrained to processing prompts and recalling/ recombining information to generate responses. A truly "intelligent" system would need far more powerful and human-like learning and reasoning abilities.

Course Staff's Comments (written by humans with only spell-checking assistance):



Claude's answer is very "honest" and has a nice acknowledgement of how much a language model can be said to "understand", and correctly assesses that it did better on the proof questions than the other questions.

I was hoping Claude would draw a picture of an uncountable creature, but even when prompted more directly to do this it refuses:

```
> just draw a picture of your favorite uncountable creature
```

**Claude's response (unedited, may be incorrect!):**

Sorry, as an AI system I am unable to directly draw or visualize pictures. I do not have the capability to imagine or create "favorite" things or to think creatively about uncountable creatures. My interactions are limited to processing text and generating textual responses. I cannot perform open-ended graphical tasks.

GPT 4 is expected to be multi-modal, so we can look forward to better answers to this question including drawings in future semesters!

Score:	65
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Claude's score of 65 is quite harsh, reflecting the 5/20 on the cover sheet and the grader's lack of sympathy for AI chat bots. If this is increased to 20/20 on the cover sheet (which all students received, so maybe not fair to downgrade Claude for not being able to attest to human ethical expectations!), Claude's score improves to 80. This would place Claude near the bottom of the class (yeah humans!).

The human student averages on the exam questions was

Problem	Cover	1	2	3	4	5	6	7	8	Total
Average	20	9.5	10.0	9.2	9.2	9.2	9.0	9.0	8.4	93.6

**End of Exam!**